Physical Cyclic Animations

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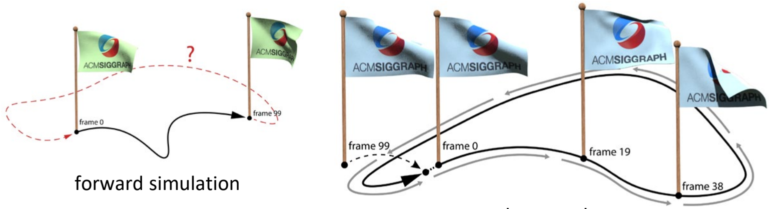
PROBLEM

Given initial values and simulation time, we aim at producing cyclic animations of physical systems. The animation should locally approximate the physical laws of motion, and globally loop back seamlessly.



forward simulation

cyclic animation

FORMULATION

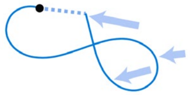
Suppose the trajectory of a moving object is guided by motion equation: $\mathrm{M}\ddot{q}(t) = f(q)$.

Keyframe Control

As a special case of key frame animation, previous methods solve this problem by adding minimum amount of control force to the system so that the status of last frame matches that of the first

$$\min_u \int_0^T \frac{1}{2}\|u\|^2_{\mathrm{M}^{-1}}\, dt$$

$$\text{s.t.} \begin{cases} \mathrm{M}\ddot{q}(t) = f(q) + u \\ q(0) = q(T) = q_0^* \\ \dot{q}(0) = \dot{q}(T) = v_0^* \end{cases}$$



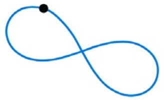
This formula requires solving a nonlinear constrained optimization.

Unconstrained Trajectory Optimization

We directly optimize the trajectory in a periodic time domain, therefore turn the problem into a unconstrained optimization in affine space:

$$\min_q \oint \frac{1}{2}\|\mathrm{M}\ddot{q}(t) - f(q)\|^2_{\mathrm{M}^{-1}}\, dt$$

$$\text{s.t.}\, q(0) = q_0^*, \dot{q}(0) = v_0^*$$



METHOD

After discretization, the discrete time domain is endowed with modular arithmetic:

$$\min_q \sum_{i=0}^{N-1} \frac{1}{2}\left\|\mathrm{M}\frac{q_{(i+1)\bmod N} - 2q_i + q_{(i-1)\bmod N}}{h^2} - f(q_i)\right\|^2_{\mathrm{M}^{-1}}$$

$$\text{s.t. } q_0 = q_0^*, q_1 = q_1^*$$

The constraint is handled by either solving the problem in the affine space or adding mass-weighted penalty terms.

Observing that the objective function is a sum of squares, we solve the optimization with **Gauss-Newton method**.

Fast projection as initial guess

We solve the key-frame control problem using fast projection to provide a good initial guess. Starting from a forward simulation with zero control force, fast projection linearizes the constraint and projects the control force to the linearized subspace at every iteration.

$$\min_u \frac{1}{2}\|u - u^{(k)}\|^2_{M^{-1}}$$

$$\text{s.t. } u \in H^{(k)} = \{u | C(u^{(k)}) + \nabla C|_{u^{(k)}}(u - u^{(k)}) = 0\}$$

ALGORITHM

Input: looping frame q_0^*, q_1^*, step size h, and total frame N

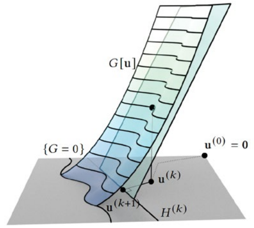
fast projection

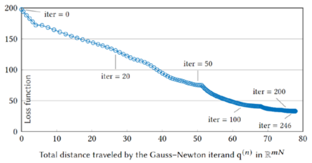
$u^{(0)} \leftarrow 0$
while $|\delta u| > \epsilon_1$:
  $q^{(k)} \leftarrow \text{forward\_sim}(u^{(k)})$
  $u^{(k+1)} \leftarrow u^{(k)} - \frac{C(u^{(k)})}{\nabla C|_{u^{(k)}} \nabla C|^T_{u^{(k)}}} \nabla C|^T_{u^{(k)}}$

Gauss-Newton

$q^{(0)} \leftarrow \text{forward\_sim}(u)$
while $|\delta q| > \epsilon_2$:
  evaluate gradient g and Hessian approximation $\widehat{H}$
  $\delta q \leftarrow -\widehat{H}^{-1} g$
  $q^{(k+1)} \leftarrow q^{(k)} + \alpha\delta q$ // α is given by line search

Output: cyclic animation q





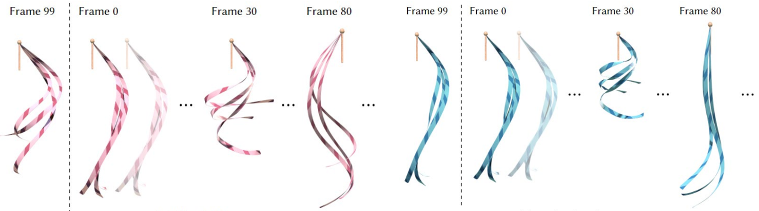
RESULTS

N-body system

Physical

Cyclic (ours)

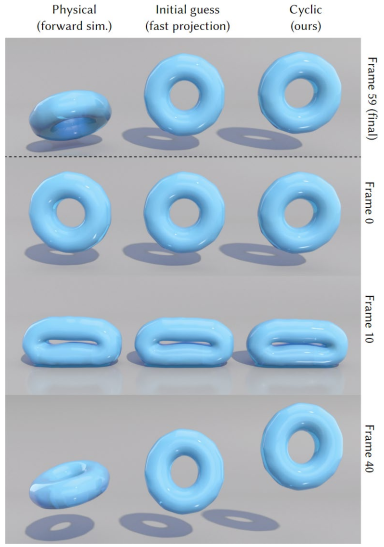
Cloth Animation



(a) Physical

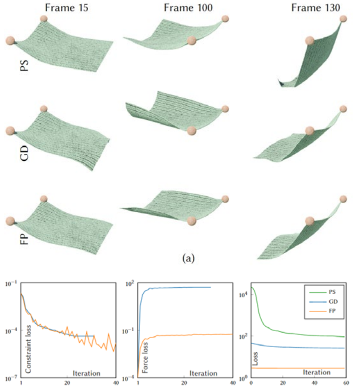
(b) Cyclic (ours)

Soft body Animation



EVALUATION

Fast projection vs. Gradient descent



convergence with iteration

Gauss-Newton with different initial guess

FUTURE WORK

- Increase scalability & acceleration: spectral method
- Energy conservation: match kinetic energy in forward simulation
- Self collision: differentiable collision model, Incremental Potential Method (IPC)